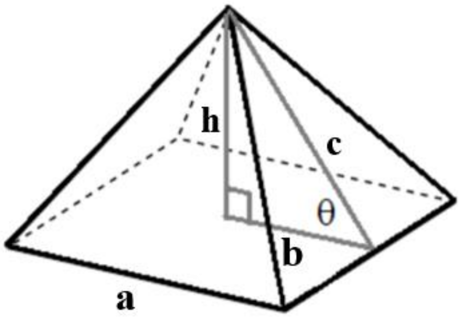
SL Paper 1 Mock A 2020 - MARKSCHEME v1

Section A

1.



$h = 3\sqrt{3}$ **A1**

$b = 3$ **A1**

$c = 6$ **A1**

Recognising that $\theta = \sin^{-1}\left(\frac{h}{c}\right)$ **(M1)**

$\theta = 60°$ $\left[\text{or } \theta = \frac{\pi}{3}\right]$. **A1**

2. Use of the formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$ **(M1)**

$P(B) = \frac{2}{3}$ **A1**

$P(A) = \frac{2}{5}$ **A1**

Recognising that $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ **(M1)**

$P(A\cup B) = \frac{13}{15}$ **A1**

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3. (a) $m=10b+a$ **A1**

(b) $n-m=9a-9b$ **A1**

Dividing both sides by 9 **(M1)**

Reasoning that $\frac{n-m}{9}=a-b\in\mathbb{Z}$ **R1**

Therefore, $n-m$ is divisible by 9. **AG**

4. Recognising that $h(x)=\int x\left(1-x^2\right)^{\frac{1}{2}}\,\mathrm{d}x$ **(M1)**

Attempting to solve by substitution **(M1)**

$h(x)=-\int\frac{u^{\frac{1}{2}}}{2}\mathrm{d}u$ **(A1)**

$h(x)=-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3}+C$ **(A1)**

Substituting both x and y values into their integrated expression including C

$\frac{2}{3}=-\frac{1}{3}+C$

$C=1$ **A1**

$h(x)=-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3}+1$ $\left[\text{or } -\frac{1}{3}\sqrt{\left(1-x^2\right)^3}+1\right]$ **A1**

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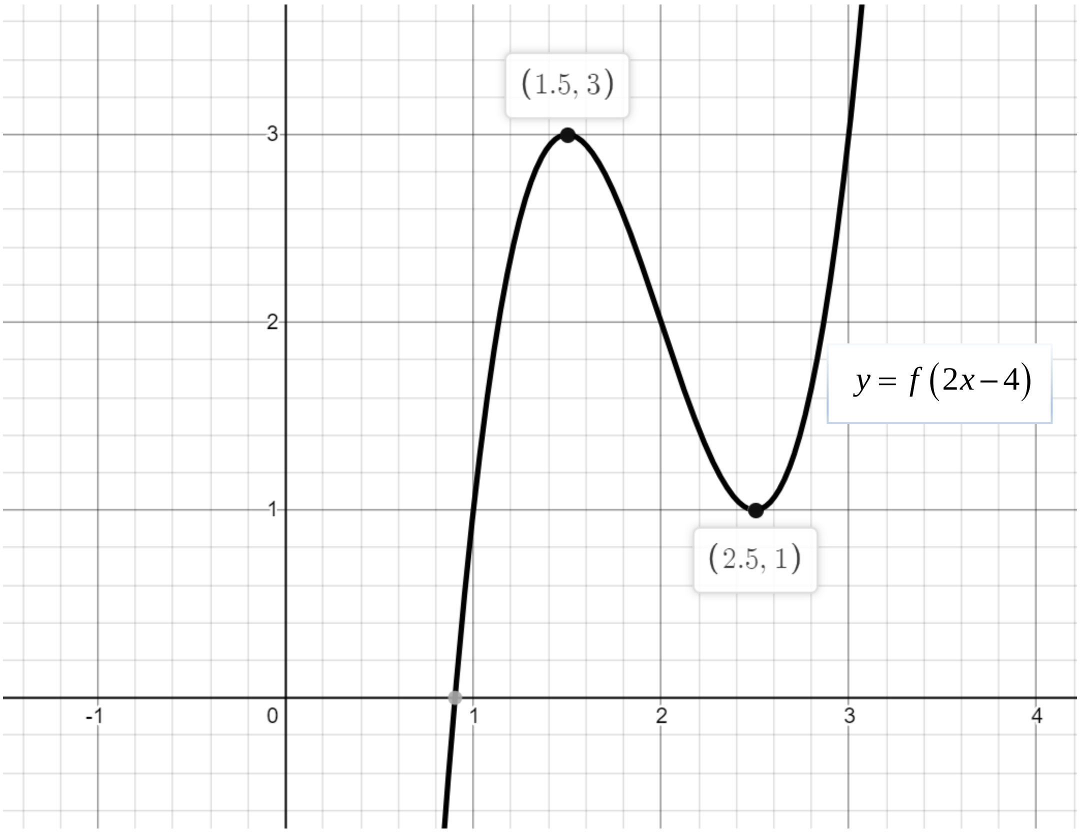
5. Recognising that the graph of $y = f\left[2(x-2)\right]$ is formed by a sequence of two transformations of the graph of $f(x)$ **(M1)**

Correctly plotting the graph of $f(x)$ with a horizontal shrink (with respect to the y axis) by a factor of $\frac{1}{2}$, followed by a horizontal translation 2 units to the right **M2**



Note: Award **A1** for each maxima/minima correctly labelled.

A1A1

6. (a) Using laws of logarithms to find $\ln\left(x(x-2)\right) = \ln(x+4)$ **(M1)**

$x = 4,\ x = -1$ **(A1)**

Verifying solutions in original equation **(M1)**

$x = 4$ **A1**

(b) Using laws of logarithms to find $\log_3\left(4x^2 - 5x - 6\right) = \log_3\left(3x^2\right)$ **(M1)**

$x = 6,\ x = -1$ **(A1)**

Verifying solutions in original equation **(M1)**

$x = 6$ **A1**

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Section B

7. (a) $f'(x) = \cos x - \sin x$ **A1A1**

(b) (i) Recognising that, at maxima/minima, $f'(x) = 0$ **(M1)**

Attempting to solve $f'(x) = 0$ **(M1)**

At maxima/minima, $x = \frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}$ **A1**

Recognising that point A has x-coordinate $x = \frac{\pi}{4}$ **(M1)**

$f\left(\frac{\pi}{4}\right) = \sqrt{2}$ **(A1)**

$A(p, q) = A\left(\frac{\pi}{4}, \sqrt{2}\right)$ **A1**

(ii) $f''(x) = -\sin x - \cos x$ **A1A1**

Recognising that, if A is a maximum, then $f''\left(\frac{\pi}{4}\right) < 0$ **(M1)**

$f''\left(\frac{\pi}{4}\right) = -\sqrt{2} < 0$ **A1**

Hence, A is a maximum. **AG**

(c) x-coordinate of B is $x = \frac{5\pi}{4}$ **A1**

Substituting $x = \frac{5\pi}{4}$ into $f(x)$ to obtain y-coordinate of B **M1**

$B(x, y) = B\left(\frac{5\pi}{4}, -\sqrt{2}\right)$ **A1**

(d) $r = \sqrt{2}$ **A1**

$c = \frac{\pi}{4}$ **A1**

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8. (a) $\mathrm{P}(2 \text{ red balls}) = \frac{2}{5}$ **A1**

$\mathrm{P}(2 \text{ yellow balls}) = \frac{1}{15}$ **A1**

Recognising that $\mathrm{P}(1 \text{ red and } 1 \text{ yellow}) = \mathrm{P}(\mathrm{RY}) + \mathrm{P}(\mathrm{YR})$ **(M1)**

$\mathrm{P}(\mathrm{RY}) = \frac{4}{6} \cdot \frac{2}{5} = \frac{4}{15}$, $\mathrm{P}(\mathrm{YR}) = \frac{2}{6} \cdot \frac{4}{5} = \frac{4}{15}$ **(A1)**

$\mathrm{P}(1 \text{ red and } 1 \text{ yellow}) = \frac{8}{15}$ **A1**

(b) $\mathrm{P}(2 \text{ red} \cap \mathrm{A}) = \frac{1}{30}$ **A1**

$\mathrm{P}(2 \text{ red} \cap \mathrm{B}) = \frac{4}{15}$ **A1**

$\mathrm{P}(2 \text{ red}) = \mathrm{P}((2 \text{ red} \cap \mathrm{A}) \cup (2 \text{ red} \cap \mathrm{B})) = \frac{3}{10}$ **A1**

(c) Recognising that a 1 or 6 rolled corresponds to bag A being chosen, and so the required probability is $\mathrm{P}(\mathrm{A} \mid 2 \text{ red})$ **(M1)**

Attempting to use the formula $\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})}$ **(M1)**

Correctly substituting $\mathrm{P}(2 \text{ red}) = \frac{3}{10}$ and $\mathrm{P}(2 \text{ red} \cap \mathrm{A}) = \frac{1}{30}$ into the formula for $\mathrm{P}(\mathrm{A} \mid 2 \text{ red})$ **M1**

$\mathrm{P}(\mathrm{A} \mid 2 \text{ red}) = \frac{1}{9}$ **A1**

9. (a) Attempting to find $g'(x)$ using the product rule **(M1)**

$g'(x) = \mathrm{e}^{-x^2} - 2x^2\mathrm{e}^{-x^2}$ **A1**

Attempting to solve $g'(x) = 0$ **(M1)**

[9. Continued on next page]

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$g'(x)=0$ where $x=\dfrac{\sqrt{2}}{2}$, therefore the x-coordinate of P is $x=\dfrac{\sqrt{2}}{2}$ **A1**

Showing that $g'(x)>0$ for $x<\dfrac{\sqrt{2}}{2}$ and $g'(x)<0$ for $x>\dfrac{\sqrt{2}}{2}$ **(M1)**

Hence, $\mathrm{P}\left(\dfrac{\sqrt{2}}{2}, y\right)$ is the one maximum on g. **AG**

(b) Attempting to find $g''(x)$ using the product rule **(M1)**

$g''(x)=\left(4x^3-6x\right)\mathrm{e}^{-x^2}$ **A1**

Inflexion points exist where $g''(x)=0$ or $g''(x)$ is undefined;
$g''(x)$ is defined for all x **(M1)**

Attempting to solve $g''(x)=0$ **(M1)**

$g''(x)=0$ for $x\geq 0$ where $x=\sqrt{\dfrac{3}{2}}$ **(A1)**

Showing that $g''(x)<0$ for $x<\sqrt{\dfrac{3}{2}}$ and $g''(x)>0$ for $x>\sqrt{\dfrac{3}{2}}$ **M1**

Hence $g(x)$ has an inflexion point $\mathrm{Q}\left(\sqrt{\dfrac{3}{2}}, y\right)$ **AG**

(c) (i) $x>\sqrt{\dfrac{3}{2}}$ **A1**

(ii) $0\leq x<\sqrt{\dfrac{3}{2}}$ **A1**

(d) Recognising that $\int_0^k x\mathrm{e}^{-x^2}\,\mathrm{d}x=\dfrac{1}{2}-\dfrac{1}{2\mathrm{e}^4}$ **(M1)**

Attempting to integrate $\int_0^k x\mathrm{e}^{-x^2}\,\mathrm{d}x$ using integration by substitution **(M1)**

$\int_0^k x\mathrm{e}^{-x^2}\,\mathrm{d}x=\int_0^k-\dfrac{\mathrm{d}u}{2}$ **(A1)**

$\dfrac{1}{2}-\dfrac{1}{2\mathrm{e}^{k^2}}=\dfrac{1}{2}-\dfrac{1}{2\mathrm{e}^4}$ **(A1)**

$k=2$ **A1**